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Characterisation of viscoelastic layers in sandwich panels via an inverse technique

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ABSTRACT

A new inverse technique to characterise the nonlinear mechanical properties of viscoelastic core layers in sandwich panels has been developed based on simple vibration tests. The present methodology allows one to preserve the frequency and temperature dependences of the storage and loss moduli of viscoelastic materials for a wide range of frequencies and to perform a structural analysis using high damping tests. The computational effort has been substantially reduced by using an optimisation based on the planning of the experiments and the response surface technique in order to minimise the error functional. This new inverse technique has been tested and applied to characterise the viscoelastic properties of a 3M damping polymer (ISD-112) used as a core material in sandwich panels. The identified material properties have been verified successfully by comparing structural dynamic parameters obtained from the numerical analysis with experiments.

1. Introduction

Over the last three decades, considerable efforts have been made to study the mechanical properties of advanced composite materials due to the need to characterise their performance and reliability for safety requirements. To do so, many different methods have been developed. For many orthotropic sheet materials, one of the simplest methods is based on low frequency vibrations [1–3]. This method is not only the simplest approach but also the only measurement that provides good predictions for the investigated range of frequencies. Two other general methods might be used, namely static measurements [4–6] and ultrasonics [7–9]. However, neither of them is fully appropriate to characterise the mechanical properties of advanced composite materials. These techniques are particularly limited for determining damping "constants", as these parameters are expected to be frequency- and temperature-dependent.

Only a few researchers have measured the viscoelastic properties of sandwich composite materials using an inverse technique based on vibration tests. For instance, both forced steady-state harmonic vibrations and free vibrations have been used in Ref. [10] in order to characterise the constitutive parameters of the Voigt model and three parameters model to describe the uniaxial viscoelasticity of sandwich composite beams. These beams consist of a core made of polyvinyl chloride foam and two outer laminate layers made of glass fibre-reinforced polyester. To identify material parameters of aluminium honeycomb sandwich panels, an orthotropic Timoshenko beam model has been used in Ref. [11], where the elastic constants and modal damping ratios have been determined by minimising the error between the experimental and

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analytical results. Ref. [12] proposes an inverse method based on flexural resonance frequencies using the sandwich beam theory for finite element modelling. However, the material loss factors measured in these studies are either too low, constant or they have only been determined in a narrow frequency range.

For these reasons, the current study is focused on the development of a new inverse technique based on simple vibration tests in order to characterise the nonlinear mechanical properties of various viscoelastic materials widely used in sandwich composite applications. This novel approach will allow to analyse sandwich structures with high damping properties and to characterise their viscoelastic properties across a wide range of frequencies, where the storage and loss moduli depend on frequency and temperature. This new technique has been successfully tested by characterising the viscoelastic properties of a 3M damping polymer (ISD-112) used as a core material in sandwich panels with aluminium faces.

2. Inverse technique

The inverse technique developed in this study is shown in Fig. 1. It uses vibration tests and consists of the experimental set-up, the numerical model and the material parameters identification procedure developed by applying a non-direct optimisation method based on the planning of the experiments and a response surface method in order to considerably decrease the computational efforts [13]. The first step involves the planning of the investigation depending on the number of measured parameters and experiments. Next, a finite element analysis is applied at the reference points of the experimental design and the different dynamic parameters of the structure are calculated. In the third step of this technique, these numerical data are used to determine simple functions using a response surface method. Simultaneously, vibration experiments are carried out to measure the natural resonance frequencies and corresponding loss factors of the viscoelastic structures. The identification of the material properties is performed in the final step of the method by minimising the error functional, which describes the difference between the experimental and numerical parameters of the structural responses.

2.1. Experimental analysis

The experimental set-up used for the vibration testing of the sandwich panels is shown in Fig. 2. The impulse technique consists of an excitation generated by a PCB impulse hammer equipped with a built-in force transducer. The structural response is measured using three accelerometers located on the panel as shown in Fig. 2. Both the input and output signals are converted inside a signal analyser into the frequency domain using a fast Fourier transformation, and the frequency response functions are generated. Then, these functions are transferred to the modal analysis program, where the natural resonant frequencies and corresponding loss factors are calculated.

2.2. Finite element analysis

In the presented inverse technique, the finite element method is used for modelling and dynamic analysis of sandwich panels having viscoelastic core layers.

2.2.1. Finite element model

The finite element modelling is based on the first-order shear deformation theory, including rotation around the normal. In this theory, the well-known expressions for the displacements have the following form:

$$u = u_0 + z\gamma_x, \quad v = v_0 + z\gamma_y, \quad w = w_0 \tag{1}$$

where u_0, v_0, w_0 are the displacements in the reference plane, z the coordinate of the point of interest in the reference plane, and γ_x, γ_y are the rotations connected with the transverse shear deformations. In the case of sandwiches, this hypothesis is applied separately for each layer (Fig. 3). It corresponds to the broken line model [14] and satisfies the

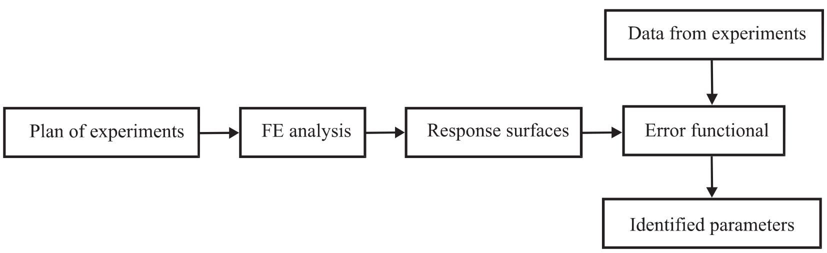

Fig. 1. Inverse procedure.

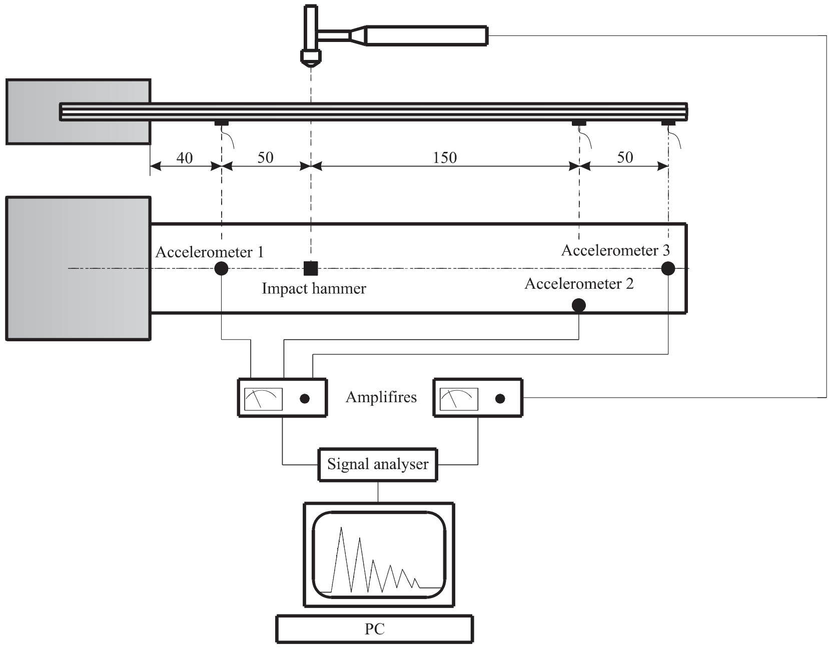

Fig. 2. Experimental set-up.

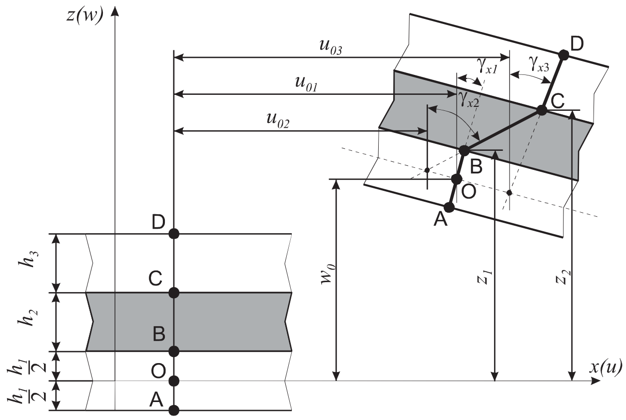

Fig. 3. Kinematic assumptions for a sandwich plate.

following displacement continuity conditions between the different layers

$$u^{(1)} = u^{(2)}|_{z=z_1}, \quad u^{(2)} = u^{(3)}|_{z=z_2}$$

$$v^{(1)} = v^{(2)}|_{z=z_1}, \quad v^{(2)} = v^{(3)}|_{z=z_2}$$

$$w^{(1)} = w^{(2)}|_{z=z_1}, \quad w^{(2)} = w^{(3)}|_{z=z_2} \tag{2}$$

where the number of each layer is indicated in the brackets.

2.2.2. *Dynamic analysis*

To describe the rheological behaviour of the viscoelastic materials, the complex modulus representation [15] is applied. Using this model, the constitutive relations can be expressed in the frequency domain as follows:

$$\sigma_0 = E^*(\omega)\varepsilon_0 = E(\omega)[1 + \mathrm{i}\eta(\omega)]\varepsilon_0, \quad \eta(\omega) = \frac{E''(\omega)}{E(\omega)} \tag{3}$$

where σ_0 and ε_0 represent the amplitude of the harmonically time-dependent stress and strain, respectively, E^* the complex modulus of elasticity, E, E'' are the real and imaginary parts of the complex modulus of elasticity, η the loss factor and ω represents the frequency. It is necessary to note that the storage and loss moduli depend on the frequency in this case.

Using the method of complex eigenvalues, the damped eigenfrequencies and corresponding loss factors are determined from the free vibration analysis of the structure according to

$$[\mathbf{K}^*(\omega) - \omega^{*2}\mathbf{M}]\bar{\mathbf{X}}^* = 0 \tag{4}$$

where $\mathbf{M}$ is the mass matrix of the structure, $\mathbf{K}^*(\omega) = \mathbf{K}(\omega) + \mathrm{i}\mathbf{K}''(\omega)$ the complex stiffness matrix of the structure, and $\omega^* = \omega + \mathrm{i}\omega''$ the complex eigenfrequency. The real part ω represents the damped eigenfrequency of the structure and the imaginary part ω'' specifies the rate of decay of the dynamic process. The matrix $\mathbf{K}(\omega)$ is determined using the storage moduli $E(\omega)$ and $G(\omega)$, while $\mathbf{K}''(\omega)$ is calculated using the imaginary parts of the complex moduli $E''(\omega) = \eta_E(\omega)E(\omega)$ and $G''(\omega) = \eta_G(\omega)G(\omega)$, where $\eta_E(\omega)$ and $\eta_G(\omega)$ represent the material loss factors.

Eq. (4) can be written as a nonlinear generalised eigenvalue problem

$$\mathbf{K}^*(\omega)\bar{\mathbf{X}}^* = \lambda^*\mathbf{M}\bar{\mathbf{X}}^* \tag{5}$$

where $\lambda^* = \omega^{*2}$ is a complex eigenvalue and $\bar{\mathbf{X}}^*$ the complex eigenvector. The solutions of Eq. (5) start at a constant frequency ($\omega = \text{const}$). Then, for each step, the linear generalised eigenvalue problem with $\mathbf{K}^*(\omega) = \text{const}$ is solved using the Lanczos method [16], which is programmed in a truncated version, where the generalised eigenvalue problem is transformed into a standard eigenvalue problem with a reduced order symmetric three diagonal matrix. Orthogonal projection operations are used with greater economy and elegance using elementary reflection matrices. An iteration process terminates, when the following condition is satisfied:

$$\frac{|\omega_{i+1} - \omega_i|}{\omega_i} * 100\% \le \xi \tag{6}$$

where ξ is a desired precision and ω_{i+1} the real part of the eigenfrequency of the structure calculated from the linear generalised eigenvalue problem for the storage and loss moduli at the frequency ω_i, which were obtained from the same equation in the previous step. The modal loss factors of the structure for each vibration mode are determined using the following equation:

$$\eta_n = \frac{\lambda_n''}{\lambda_n} \tag{7}$$

This approach allows to preserve the frequency dependence of the viscoelastic materials and to analyse structures with a high damping. Other approaches than those used in the inverse procedure, namely frequency and transient response analyses [17], are applied to validate the identified material properties.

2.3. Material identification procedure

The main idea of the material identification procedure based on vibration tests and an indirect optimisation method is that simple mathematical models (response surfaces) are determined only using the finite element solutions for the reference points of the plan of experiments. The identification parameters for all eigenfrequencies in the examined frequency range are obtained by minimising the error functional, which describes the difference between the measured and numerically calculated parameters of the structural responses. The calculation of the identification functional is significantly simplified in this case compared to conventional optimisation methods. Indeed, one design space only is necessary to identify the viscoelastic material properties in the desired frequency range for different temperatures.

2.3.1. Planning of experiments

Let us consider a criterion for the planning of the experiments that is independent on the mathematical model chosen for the designed object or process. The initial information for the development of the plan is the number of factors n and the number of experiments k. The points of experiments in the domain of factors are distributed as regularly as possible. For this reason the following criterion is used:

$$\Phi = \sum_{i=1}^{k}\sum_{j=i+1}^{k}\frac{1}{l_{ij}^2} \Rightarrow \min \tag{8}$$

where l_{ij} is the distance between the points numbered i and j ($i \ne j$). Physically, it is equal to the minimum of the potential energy of the repulsive forces for the points with unit mass if the magnitude of these repulsive forces is inversely proportional to the distance between the points.

For each number of factors n and for each number of experiments k, it is possible to determine a plan of experiments, but it requires a considerable computing time. Therefore, each plan of experiment is only determined once and can be used for various design situations. Each plan of experiments is characterised by the matrix of plan B_{ij}. The domain of factors is determined as $x_j \in [x_j^{\min}, x_j^{\max}]$ and the points of experiments are calculated using the following expression:

$$x_j^{(i)} = x_j^{\min} + \frac{1}{k-1}(x_j^{\max} - x_j^{\min})(B_{ij} - 1), \quad i = 1,2,\ldots,k, \; j = 1,2,\ldots,n \tag{9}$$

2.3.2. Response surface method

In the present study, the form of the regression equation is a priori unknown. There are two requirements for the regression equation: accuracy and reliability. Accuracy is characterised by the minimum of the standard deviation of the table data from the values given by the regression equation. By increasing the number of terms in this equation, it is possible to obtain a complete agreement between the table data and the values obtained from the regression equation. However, it is necessary to note that the predictions in the intervals between the points of the table could be not so good. To improve the accuracy of the predictions, it is necessary to decrease the distance between the points of experiments by either increasing the number of experiments or decreasing the domain of factors. The reliability of the regression equation can be characterised by assuming that the standard deviations for the points of the table and for any other points are approximately the same. Obviously, the reliability is greater for a smaller number of terms in the regression equation.

The regression equation can be written in the following form:

$$y = \sum_{i=1}^{p} A_i f_i(x_j) \tag{10}$$

where A_i are the coefficients of the regression equation and $f_i(x_j)$ are the functions from the pool of simple functions $\theta_1, \theta_2, ..., \theta_m$ which are assumed to satisfy,

$$\theta_m(x_j) = \prod_{i=1}^{s} x_j^{\xi_{mi}} \tag{11}$$

where ξ_{mi} is a positive or negative integer, including zero. The derivation of the equation from the pool of simple functions is carried out in two steps: the selection of the perspective functions from the pool and a step-by-step elimination of the selected functions.

In the first step, all variants are tested using a least square method and the function, that leads to a minimum of the sum of the deviations, is chosen for each variant. In the second step, the elimination is carried out using the standard deviation defined as

$$\sigma_0 = \sqrt{\frac{S}{k-p+1}}, \quad \sigma = \sqrt{\frac{1}{k-1}\sum_{i=1}^{k}\left(y_i - \frac{1}{k}\sum_{j=1}^{k} y_j\right)^2} \tag{12}$$

or the correlation coefficient given by

$$c = \left(1 - \frac{\sigma}{\sigma_0}\right) * 100\% \tag{13}$$

where k is the number of experimental points, p the number of selected prospective functions, and S the minimum of the sum of the deviations. It is more convenient to characterise the accuracy of the regression equation using the correlation coefficient (see Fig. 4) since it is particularly sensitive to the choice of selected functions. If insignificant functions are eliminated from the regression equation, the reduction of the correlation coefficient is negligible. If only significant functions are present in the regression equation, eliminating one function leads to an important decrease of the correlation coefficient.

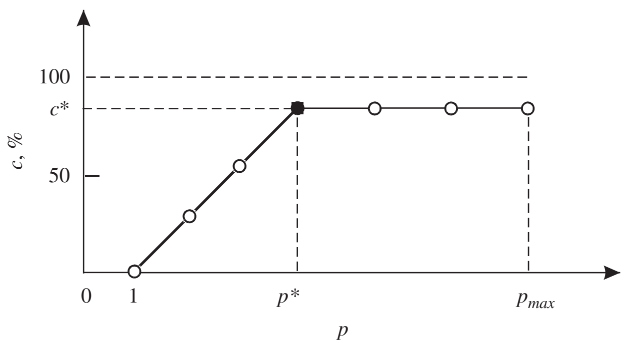

Fig. 4. Diagram of elimination for the correlation coefficient.

2.3.3. Error functional minimisation

In the case of the identification of viscoelastic material properties, the error functional, which describes the difference between the experimental and numerical parameters of the structural responses, can be written for each eigenfrequency as

$$\Phi_n(x) = \frac{(f_n^{EXP} - f_n^{FEM})^2}{(f_n^{EXP})^2} + \frac{(\eta_n^{EXP} - \eta_n^{FEM})^2}{(\eta_n^{EXP})^2} \Rightarrow \min \tag{14}$$

To minimise the error functional, the following constrained nonlinear optimisation problem must be solved:

$$\min \Phi(x), \quad H_i(x) \geq 0, \quad G_j(x) = 0, \quad i = 1, 2, \ldots, I, \; j = 1, 2, \ldots, J \tag{15}$$

where I and J are the numbers of the inequality and equality constraints. This problem is replaced by an unconstrained minimisation problem, in which the constraints are taken into account with the penalty functions. A new version of random search method [18] is used to solve the formulated optimisation problem. In addition, a curve fitting procedure is required to obtain the frequency dependent viscoelastic material properties for the measured temperatures.

3. Identification examples and results verification

The present inverse technique is tested and applied to characterise the viscoelastic material properties of a 3M damping polymer (ISD-112) used as a core material in sandwich panels.

3.1. Inverse technique testing

The sandwich beam shown in Fig. 5 has been chosen to test the performance of the inverse technique developed in this study. It has the following dimensions: width $B = 0.05$ m, length $L = 0.3$ m and thickness of layers $h_1 = 0.0012$ m, $h_2 = 0.0001016$ m, $h_3 = 0.0008$ m. The external layers are made out of aluminium 2024 T6 with characteristics: $E = 64$ GPa, $\upsilon = 0.32$, and $\rho = 2695\,\mathrm{N\,s^2/m^4}$. Clamped boundary conditions are applied at one side of the beam, whose dynamic characteristics such as its eigenfrequencies and corresponding loss factors (see Table 1) have been determined from numerical experiments using a complex eigenvalues method and the material properties of the 3M damping polymer ISD-112 from Ref. [15].

In order to describe the isotropic viscoelastic material properties of the beam, only one material parameter is necessary, namely the elastic modulus $E^*(\omega) = E(\omega) + iE''(\omega)$. However in this work, it is a complex value consisting of the storage $E(\omega)$ and loss $E''(\omega)$ modulus parts, which are both frequency-dependent. The Poisson ratio ($\upsilon = 0.49$) and density ($\rho = 1000\,\mathrm{N\,s^2/m^4}$) are considered as known material parameters. The limits of the identified parameters are $G = 0.2-2$ MPa and $\eta = 0-1.4$ for a frequency range of $f = 0$–2000 Hz and a temperature of 37.8 °C.

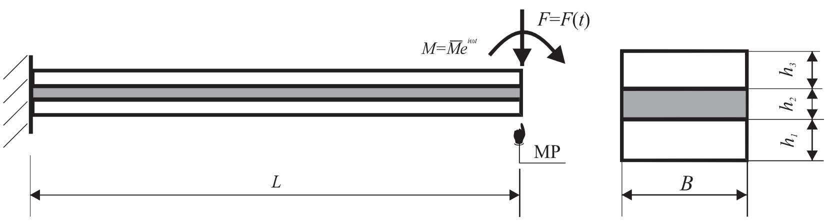

Fig. 5. Sandwich beam.

Table 1
Verification of dynamic characteristics for the sandwich beam (identification from numerical tests).

Mode n	f_n^{EXP} (Hz)	f_n^{FEM} (Hz)	Δ (percent)	η_n^{EXP}	η_n^{FEM}	Δ (percent)
1	16	16	0	0.055	0.058	5.5
2	80	80	0	0.156	0.164	5.1
3	210	212	1.0	0.206	0.214	3.9
4	391	395	1.0	0.223	0.231	3.6
5	625	631	1.0	0.223	0.230	3.1
6	908	915	0.8	0.216	0.220	1.9
7	1241	1247	0.5	0.205	0.206	0.5
8	1622	1628	0.4	0.191	0.190	0.5

The plan of experiments is determined for 2 design parameters and 48 experiments. Then, the finite element analysis is performed for the 48 experimental points and eight first dynamic characteristics are determined. Using these numerical values, the approximating functions (response surfaces) for all eigenfrequencies (Table 2) as well as the corresponding loss

Table 2
Response surfaces with correlation coefficients for eigenfrequencies.

Mode n	f_n (Hz)		c (percent)
1	$18.6-1.28z_2+\frac{1.13z_2}{z_1}+\frac{0.0329}{z_2^2}+0.592z_1z_2+\frac{0.000742}{z_1^3}-\frac{0.923}{z_1}$		94
	$z_1=0.168E$	$z_2=0.5+0.138E''$	
2	$95.9+7.14z_1-\frac{14.6}{z_1}+\frac{0.234}{z_1^2}+\frac{10.8z_2}{z_1}+\frac{2.18}{z_1z_2}$		95
	$z_1=0.168E$	$z_2=0.5+0.138E''$	
3	$220+33.3z_1-\frac{12.2}{z_1}+\frac{1.14}{z_1^2}+\frac{11.5z_2^2}{z_1}-\frac{1.25z_2}{z_1^2}$		96
	$z_1=0.168E$	$z_2=0.5+0.138E''$	
4	$262+107z_1+110z_2-\frac{8.27}{z_1}+\frac{24.3}{z_2}+\frac{0.443}{z_1^2}-52.3z_1z_2$		98
	$z_1=0.168E$	$z_2=0.5+0.138E''$	
5	$563+81.4z_1-\frac{134}{z_1}+\frac{29}{z_1z_2}+\frac{89.2z_2}{z_1}$		98
	$z_1=0.388+0.187E$	$z_2=0.5+0.138E''$	
6	$889+66.6z_1+20.8z_2-11z_1^2+6.89z_2^2$		98
	$z_1=-1.22+0.373E$	$z_2=-1.0+0.277E''$	
7	$1200+72.6z_1+18.6z_2-8.91z_1^2+7.04z_2^2$		98
	$z_1=-1.22+0.373E$	$z_2=-1.0+0.277E''$	
8	$1560+76.7z_1+16.2z_2-7.22z_1^2+6.66z_2^2$		98
	$z_1=-1.22+0.373E$	$z_2=-1.0+0.277E''$	

Table 3
Response surfaces with correlation coefficients for loss factors.

Mode n	η_n		c (percent)
1	$-0.115+\frac{0.246}{z_1}+0.102z_2-\frac{0.13z_2}{z_1}+\frac{0.000429}{z_1^2}-\frac{0.091}{z_1z_2}+\frac{0.0154}{z_2^2}$		91
	$z_1=0.168E$	$z_2=0.5+0.138E''$	
2	$0.153-\frac{0.0558}{z_2}-0.0661z_1+\frac{0.167}{z_1}-\frac{0.0864}{z_1z_2}$		94
	$z_1=0.168E$	$z_2=0.5+0.138E''$	
3	$0.0452-\frac{0.188}{z_2}-\frac{0.325}{z_1z_2}+\frac{0.66}{z_1}+\frac{0.079}{z_2^2}$		97
	$z_1=0.388+0.187E$	$z_2=0.5+0.138E''$	
4	$-0.152+0.129z_1-\frac{0.385}{z_1}-0.348z_2+\frac{1.27z_2}{z_1}-\frac{0.237z_2^2}{z_1^2}$		97
	$z_1=0.388+0.187E$	$z_2=0.5+0.138E''$	
5	$0.0927-\frac{0.0468}{z_2}-\frac{0.192}{z_1}+\frac{0.381z_2}{z_1}$		97
	$z_1=0.388+0.187E$	$z_2=0.5+0.138E''$	
6	$0.2+0.196z_2-0.0585z_1-0.0592z_1z_2$		95
	$z_1=-1.22+0.373E$	$z_2=-1.0+0.277E''$	
7	$-0.266+0.587z_2-0.154z_1+\frac{0.0815z_1}{z_2}-0.199z_2^2$		98
	$z_1=0.168E$	$z_2=0.5+0.138E''$	
8	$-0.0952+0.276z_2-\frac{0.0221}{z_2}+\frac{0.0176z_1}{z_2}-0.0671z_1z_2$		98
	$z_1=0.168E$	$z_2=0.5+0.138E''$	

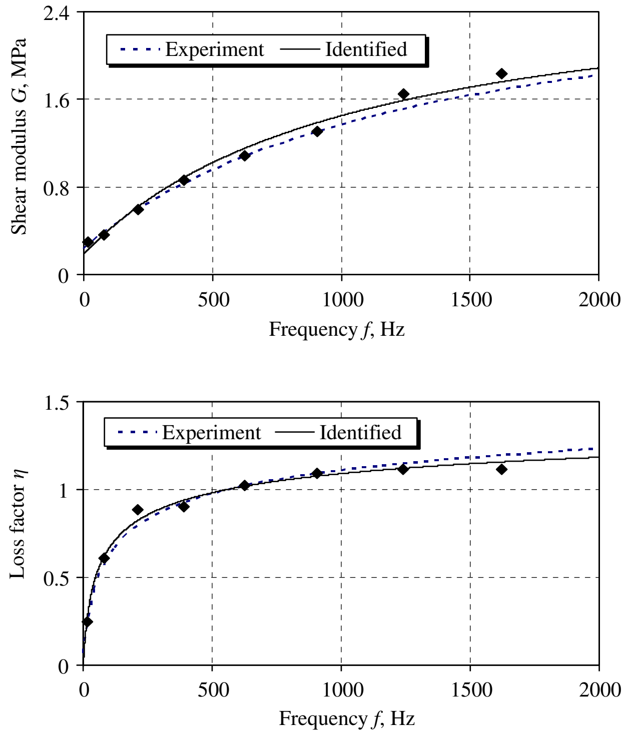

Fig. 6. Viscoelastic material properties identified from numerical tests.

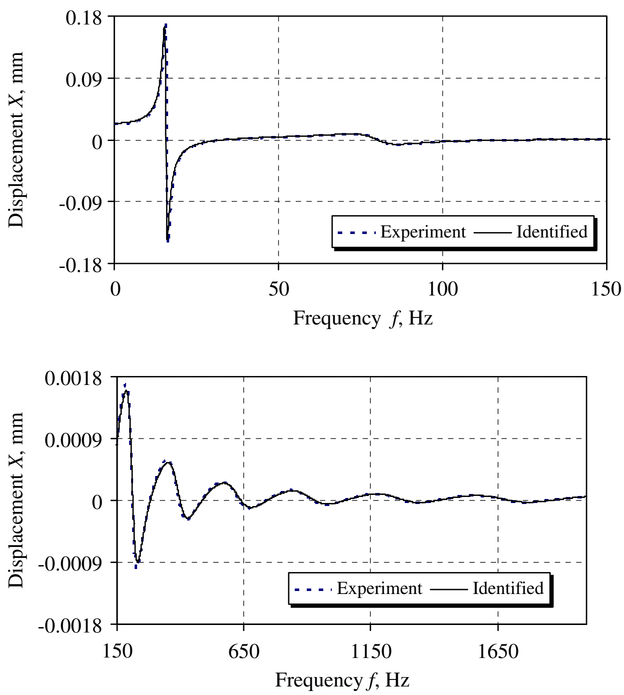

Fig. 7. Verification using a frequency response analysis.

factors (Table 3) were obtained with correlation coefficients higher than 90 percent. Minimising the error functional of Eq. (14), the material properties are found for each eigenfrequency. These values are shown in Fig. 6 for each point. Next, a curve fitting procedure was applied and the following shear modulus (MPa) and material loss factor as functions on

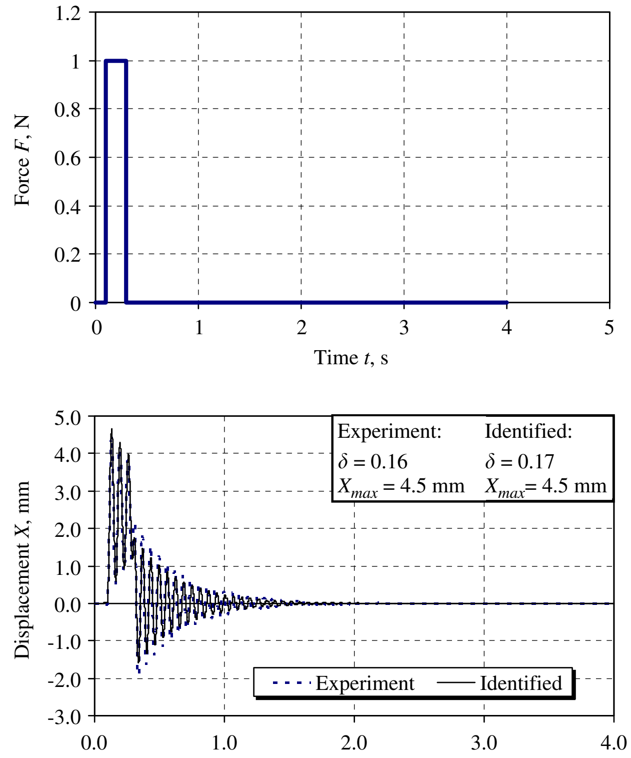

Fig. 8. Verification using a transient response analysis.

frequency were obtained:

$$G = 2.783 - 1.023/z$$

where $z = 0.394 + 0.0003736f$.

$$\eta = 1.683 + 0.001468/z - 0.5274/z^{0.25}$$

where $z = 0.005 + 0.0006134f$.

These dependencies are graphically presented in Fig. 6 as solid lines together with the graphs (stroke lines) from [15]. Fig. 6 shows a very good agreement between the identified and experimental material properties.

Additional verification was carried out using a complex eigenvalues method, a frequency response analysis by applying a unit bending moment at the end of the sandwich beam, and a transient response analysis by applying a rectangular force impulse at the end of the sandwich beam. The transverse displacements were measured at the free end of the clamped beam for both analyses. It is observed that the damped eigenfrequencies and corresponding loss factors (Table 1), the frequency (Fig. 7) and the transient (Fig. 8) responses obtained from the finite element method using the identified viscoelastic properties are in very good agreement with the experimental material properties from [15]. The difference for the eigenfrequencies is less than 1 percent in terms of residuals and less than 5 percent in most cases for the loss factors.

3.2. Inverse technique application

The structural dynamic characteristics, namely the eigenfrequencies and corresponding loss factors (Table 4), have been obtained from physical vibration experiments using an impulse technique (Fig. 2) and the same type of sandwich panel but the different thickness of the core layer $h_2 = 0.000254$ m. The limits of the identified parameters were chosen in the present analysis as follows: $G = 0.2-7.2$ MPa and $\eta = 0-1.5$. Due to their larger range, the number of experiments has been increased to 98. The material properties of the 3M viscoelastic damping polymer ISD-112 were identified for each eigenfrequency by minimising the error functional of Eq. (14). These values are shown in Fig. 9 for each point. After applying a curve fitting procedure, the following shear modulus (MPa) and material loss factor were obtained in the frequency range $f = 5$–1500 Hz:

$$G = 3.195 - 1.397/z + 4.028z^2$$

Table 4
Verification of dynamic characteristics for the sandwich beam (identification from physical experiments).

Mode n	f_n^{EXP} (Hz)	f_n^{FEM} (Hz)	Δ (percent)	η_n^{EXP}	η_n^{FEM}	Δ (percent)
1	15	16	6.7	0.22	0.20	9.1
2	86	83	3.5	0.32	0.29	9.4
3	227	223	1.8	0.34	0.34	0
4	419	420	0.2	0.38	0.36	5.3
5	663	673	1.5	0.34	0.34	0
6	983	979	0.4	0.28	0.30	7.1
7	1340	1337	0.2	0.24	0.24	0

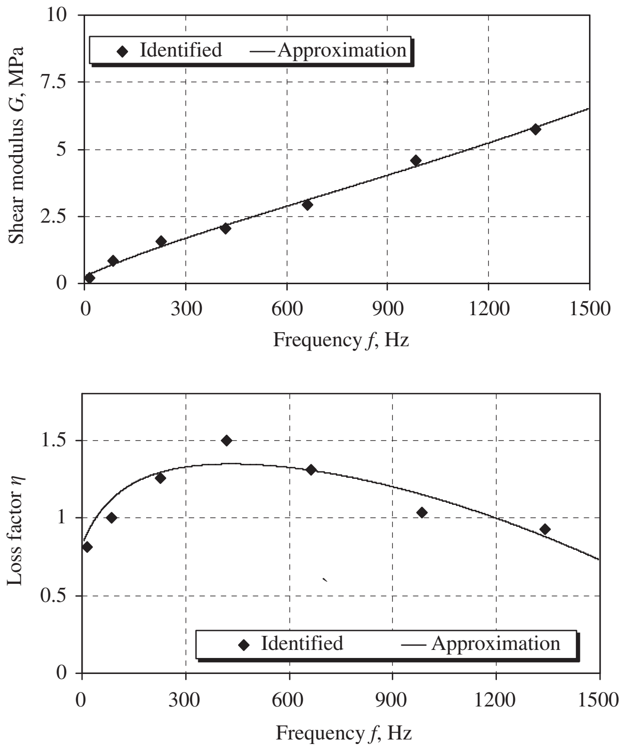

Fig. 9. Viscoelastic material properties identified from physical experiments.

where $z = 0.3932 + 0.0004528f$.

$$\eta = 1.669 - 0.8355/z - 0.7056/z^2$$

where $z = 0.1 + 0.0006716f$.

These approximated values have subsequently been used in the finite element analysis in order to verify the identified material properties by comparing the actual experimental results to those obtained numerically using the complex eigenvalues method.

Table 4 presents a good correlation between the experimental and numerical results. The largest difference is observed for the first two eigenfrequencies and corresponding loss factors. This can be explained by some inaccuracy in the experiments, namely by low values of the clamping forces at the fixed end of the sandwich beam. The measured damping values are always higher than those calculated numerically due to additional energy dissipation in the clamping device, which has not been considered during the finite element analysis. Clamped boundary conditions have been applied to avoid problems due to higher eigenfrequencies, which are damped out so well that it becomes practically impossible to measure suitable frequency responses for a structure with free–free boundary conditions.

4. Conclusions

A new inverse technique based on vibration tests has been developed to characterise the nonlinear mechanical properties of viscoelastic materials that are widely used in various sandwich composite applications. An optimisation approach based on the planning of the experiments and a response surface technique has been chosen in this work to minimise the error functional and to considerably decrease the computational efforts. The present methodology allowed to preserve the frequency and temperature dependencies of the storage and loss moduli of viscoelastic materials in a wide range of frequencies and to analyse structures using high damping tests. This new inverse technique has been tested and successfully applied to characterise the viscoelastic material properties of a 3M damping polymer (ISD-112) used as a core material in sandwich panels. A very good agreement between experimental and numerical results was obtained. The numerical experiments have shown that the accuracy of the developed inverse technique and identified material properties only depends on the accuracy of the physical experiments. The experimental errors mainly appear to be due to badly simulated boundary conditions, an added mass from exciting devices, air damping, and measurement noise.

It is important to note that our current approach, like any other inverse approach based on vibration tests, has a non-destructive character and does not require special specimens for testing. The identified material properties of a sandwich core generally reflect all the features of the technological processes used for the production of the sandwich structures. The inverse technique presented in this work can be applied not only to characterise nonlinear viscoelastic material properties of sandwich layers, but also to identify isotropic, orthotropic, elastic or viscoelastic material properties of various composites and structures due to its universality.

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